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Superconvergence sum rules for backward meson–baryon scatterings

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Abstract. Superconvergence sum rules for the $\bar{K}N \rightarrow \bar{K}N$ elastic process and the $\bar{K}\Lambda \rightarrow \bar{K}\Sigma$ inelastic process are constructed and tested by saturating them with the low-lying intermediate states. A value for $N^*(1470)K\Lambda$ coupling is calculated.

1. Introduction

Recently much work (Dass and Michael 1967, Griffiths and Palmer 1967, Ramachandran 1968, Sharma *et al.* 1969) has been done on superconvergence sum rules for $u = 0$ meson–baryon scattering. These investigations have assumed (i) that the asymptotic behaviour of the invariant amplitude is determined by the leading Regge poles, and (ii) that the resulting superconvergence relations can be well approximated by taking contributions from a finite number of low-lying resonant states in the zero-width approximation. The aim of this paper is to investigate the $\bar{K}N$ elastic process and the $\bar{K}\Lambda \rightarrow \bar{K}\Sigma$ inelastic process within these assumptions to obtain information on B*BM couplings which are not experimentally accessible.

Any invariant amplitude $f(s)$ which obeys an unsubtracted dispersion relation and is subject to the asymptotic bound $|f(s)| \sim s^{\alpha}$, $\alpha < -1$, will obey (De Alfaro *et al.* 1966) the following superconvergence relation:

$$\int_{-\infty}^{+\infty} \operatorname{Im} f(s)\, \mathrm{d}s = 0. \tag{1}$$

For backward meson–baryon scattering processes it is known that A and B have the high-energy behaviour

$$A \sim s^{\alpha(u)-\frac{1}{2}}, \qquad B \sim s^{\alpha(u)-\frac{1}{2}} \tag{2}$$

where s is the total energy in the direct channel, u is the square of the cross momentum transfer and $\alpha(u)$ is the parameter for the leading Regge trajectory.

2. Elastic process

For the $\bar{K}N$ elastic process the quantum numbers required for the u exchange are $S = +1$ and $I = 0, 1$. Since no such particle is known we can safely assume that $\alpha_{I=0,1}(u = 0) < -\frac{1}{2}$. Therefore, we can write the following four sum rules:

$$\int_{-\infty}^{+\infty} \operatorname{Im} D^I\, \mathrm{d}s = 0 \qquad (D = A \text{ or } B;\ I = 0 \text{ or } 1). \tag{3}$$

These sum rules can be re-expressed as

$$\sum_i C_{Ii}{}^{us} \int_0^{\infty} \operatorname{Im} D_i{}^I\, \mathrm{d}s + \sum_j C_{Ij}{}^{ut} \int_0^{\infty} \operatorname{Im} D_j{}^I\, \mathrm{d}s = 0 \tag{4}$$

where the C's are the crossing matrix elements and i and j may take values 0, 1.

We saturate these sum rules with the low-lying intermediate states Λ, Σ, $Y_1^*(1385)$ in the s channel and ρ, ω, ϕ in the t channel. Then the resulting sum rules are

$$-\tfrac{1}{2}D_\Lambda+\tfrac{3}{2}D_\Sigma+\tfrac{3}{2}D_{Y_1^*}-\tfrac{3}{2}D_\rho-\tfrac{1}{2}D_\omega-\tfrac{1}{2}D_\phi = 0 \tag{5}$$

$$\tfrac{1}{2}D_\Lambda+\tfrac{1}{2}D_\Sigma+\tfrac{1}{2}D_{Y_1^*}+\tfrac{1}{2}D_\rho-\tfrac{1}{2}D_\omega-\tfrac{1}{2}D_\phi = 0 \tag{6}$$

where

$$A_\Lambda = -2(m_N-m_\Lambda)g^2_{\Lambda KN}, \qquad B_\Lambda = -2g^2_{\Lambda KN}$$

$$A_\Sigma = -2(m_N-m_\Sigma)g^2_{\Sigma KN}, \qquad B_\Sigma = -2g^2_{\Sigma KN}$$

$$A_{Y_1^*} = -2g^2_{Y_1^*KN}\{(m_{Y_1^*}+m_N)(m_K^2-\tfrac{1}{2}m^2_{Y_1^*}+\tfrac{1}{3}m_N^2+\tfrac{2}{3}E^2)$$
$$+\tfrac{1}{3}(E+m_N)(m^2_{Y_1^*}-m_N^2-m_K^2)\}$$

$$B_{Y_1^*} = +2g^2_{Y_1^*KN}(\tfrac{1}{2}m^2_{Y_1^*}+\tfrac{1}{3}m_N^2-m_K^2-\tfrac{2}{3}E^2+\tfrac{2}{3}m_N E)$$

$$A_X = -2\frac{(2m_K^2+2m_N^2-m_X^2)}{2m_N}g_{XKK}g^T_{XNN}$$

$$B_X = +4g_{XKK}(g^V_{XNN}+g^T_{XNN})$$

with X = ρ, ω or ϕ and

$$E = \frac{m^2_{Y_1^*}+m_N^2-m_K^2}{2m_{Y_1^*}}.$$

We use the following values of the coupling constants (De Swart 1963):
(i) From SU(3)

$$g^2_{\Lambda NK} = \tfrac{1}{3}(1+2\alpha)^2g^2_{NN\pi}, \qquad g^2_{\Sigma NK} = (1-2\alpha)^2g^2_{NN\pi}$$

$$g_{\rho KK} = \tfrac{1}{2}g_{\rho\pi\pi}, \qquad g_{\omega KK} = -\tfrac{1}{2}g_{\rho\pi\pi}, \qquad g_{\phi KK} = \sqrt{\tfrac{1}{2}}g_{\rho\pi\pi}$$

$$g^T_{\omega NN} = -\tfrac{1}{3}(4\alpha^T-1)g^T_{\rho NN}, \qquad g^T_{\phi NN} = \frac{\sqrt{2}}{3}(4\alpha^T-1)g^T_{\rho NN}$$

$$g^V_{\omega NN} = -\tfrac{1}{3}(4\alpha^V-1)g^V_{\rho NN}, \qquad g^V_{\phi NN} = \frac{\sqrt{2}}{3}(4\alpha^V-1)g^V_{\rho NN}.$$

(ii) From universal coupling of isovector current hypothesis

$$\frac{g^T_{\rho NN}}{g^V_{\rho NN}} = 3{\cdot}7$$

$$\frac{g^V_{\rho NN}g_{\rho\pi\pi}}{4\pi} = 1{\cdot}5$$

Here

$$\alpha^T = 0{\cdot}35, \qquad \alpha^V = 1, \qquad \alpha = \frac{F}{F+D} = 0{\cdot}29$$

and

$$\frac{g^2_{NN\pi}}{4\pi} = 14{\cdot}7.$$

If we substitute these values in the sum rules, we obtain for $m_\pi^2 g^2_{Y_1^*NK}/4\pi$ the value 0·62 from the A sum rule of equation (5), 0·157 from the B sum rule of equation (5), 0·026 from the A sum rule of equation (6) and 0·28 from the B sum rule of equation (6). The model-dependent values range from 0·063 to 0·19 (Graham *et al.* 1967). Thus we find that the predictions from the superconvergence sum rules overlap considerably the results from other calculations.

3. Inelastic process

For the $\bar{K}\Lambda \to \bar{K}\Sigma$ process the leading Regge trajectory in the u channel is N. We assume (Griffiths and Palmer 1967) that $\alpha_N(u = 0) < -\frac{1}{2}$. Then the superconvergence relations are

$$\int_{-\infty}^{+\infty} \mathrm{Im}\, D^{I=1/2}\, \mathrm{d}s = 0 \tag{7}$$

where D can be either A or B.

We saturate these sum rules by the low-lying states Ξ, $\Xi^*(1530)$ in the s channel and ρ in the t channel. The resulting sum rules are

$$-D_\Xi - D_{\Xi^*} - \frac{\sqrt{6}}{2} D_\rho = 0 \tag{8}$$

where

$$A_\Xi = +3 g_{\Xi K\Lambda} g_{\Xi K\Sigma} \{\tfrac{1}{2}(m_\Lambda + m_\Sigma) - m_\Xi\}$$

$$B_\Xi = +3 g_{\Xi K\Lambda} g_{\Xi K\Sigma}$$

$$\begin{aligned} A_{\Xi^*} = {} & +3 g_{\Xi^* K\Lambda} g_{\Xi^* K\Sigma} [(m_{\Xi^*} + \tfrac{1}{2} m_\Sigma) \\ & \times \{m_K^2 - \tfrac{1}{2} m_{\Xi^*}^2 + \tfrac{1}{3} m_\Lambda m_\Sigma + \tfrac{1}{3}(E_1 m_\Sigma - E_2 m_\Sigma) + \tfrac{2}{3} E_1 E_2\} \\ & + \tfrac{1}{2} m_\Lambda (m_K^2 - \tfrac{1}{2} m_{\Xi^*}^2 + \tfrac{2}{3} E_1 E_2) \\ & - \tfrac{1}{3}\{E_1 + m_\Lambda)(m_K^2 - m_{\Xi^*}^2 + \tfrac{1}{2} m_\Lambda m_\Sigma) + \tfrac{1}{2} E_2 m_\Lambda^2\}] \end{aligned}$$

$$B_{\Xi^*} = -\tfrac{1}{2} g_{\Xi^* K\Lambda} g_{\Xi^* K\Sigma} (-2E_1 E_2 + m_\Sigma E_1 + m_\Lambda E_2 - 3 m_K^2 + \tfrac{3}{2} m_{\Xi^*}^2 + m_\Lambda m_\Sigma)$$

$$A_\rho = -2 \frac{(2 m_K^2 + m_\Lambda^2 + m_\Sigma^2 - m_\rho^2)}{m_\Lambda + m_\Sigma} g_{\rho KK} \overset{\mathrm{T}}{g}_{\rho\Lambda\Sigma}$$

$$B_\rho = +4 g_{\rho KK} (\overset{\mathrm{V}}{g}_{\rho\Lambda\Sigma} + \overset{\mathrm{T}}{g}_{\rho\Lambda\Sigma})$$

with

$$E_1 = \frac{m_{\Xi^*}^2 + m_\Lambda^2 - m_K^2}{2 m_{\Xi^*}} \quad \text{and} \quad E_2 = \frac{m_{\Xi^*}^2 + m_\Sigma^2 - m_K^2}{2 m_{\Xi^*}}.$$

We use the following SU(3) coupling constants as our input:

$$g_{\Xi K\Sigma} = \frac{1}{\sqrt{3}} g_{NN\pi}(4\alpha - 1), \qquad g_{\Xi K\Sigma} = -g_{NN\pi}$$

$$g_{\rho KK} = \tfrac{1}{2} g_{\rho\pi\pi}, \qquad \overset{\mathrm{T}}{g}_{\rho\Lambda\Sigma} = \frac{2}{\sqrt{3}} (1 - \alpha^{\mathrm{T}}) \overset{\mathrm{T}}{g}_{\rho NN}$$

$$\overset{\mathrm{V}}{g}_{\rho\Lambda\Sigma} = \frac{2}{\sqrt{3}} (1 - \alpha^{\mathrm{V}}) \overset{\mathrm{V}}{g}_{\rho NN} = 0.$$

On substituting these values in equation (8) we get for $g_{\Xi^*\Sigma K} g_{\Xi^*\Lambda K}/4\pi$ the values 0·73 and 2·02 from the A and B sum rules, respectively. Model-dependent values range from 2.62 to 8.62. Thus we again find that the calculated values considerably overlap the model-dependent values.

We can exploit such a sum rule for the $\pi N \rightarrow K\Lambda$ inelastic process to obtain information for the N*(1470)KΛ coupling. We obtain the result $g_{N^*N\pi}g_{N^*K\Lambda}/4\pi = -7{\cdot}73$ from the sum rule for A and $-5{\cdot}39$ from the sum rule for B. If we use $g_{N^*N\pi}/4\pi = 1{\cdot}33$ from decay ($\Gamma(N^* \rightarrow N\pi) = 143$ MeV (Rosenfeld *et al.* 1969)) we predict for $g_{N^*K\Lambda}/4\pi$ a value $-5{\cdot}81$ from the sum rule for A and $-4{\cdot}02$ from the sum rule for B. Since both the predicted values are close to each other, we can safely say that $g_{N^*K\Lambda}/4\pi$ should be of the order of -5 for the sum rules to be satisfied.

4. Discussion

Our results should not be taken too seriously because of the uncertainties involved in the SU(3) values of the couplings used by us. Nevertheless, our calculations show that there is enough scope of practical utilization of superconvergence sum rules to predict coupling constants which are not amenable to experiments. This is possible because of the assumption that the dominant contributions are obtained from the low-lying intermediate states. Our calculations support this assumption.

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